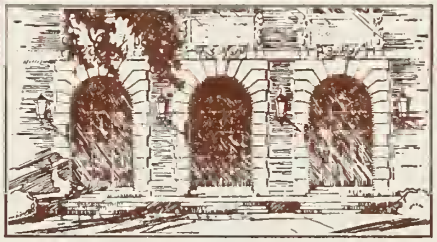

Center for Advanced Computation

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
URBANA, ILLINOIS 61801

CAC Document No. 64

COMPUTATIONAL MATHEMATICS ABSTRACTS

Edited By

Geneva Belford, Jonathan Lermit,
George Purdy, and Ahmed Sameh

February 1973

CAC Document No. 64

Computational
Mathematics Abstracts

Edited by

Geneva Belford
Jonathan Lermit
George Purdy
Ahmed Sameh

Applied Mathematics Group
Center for Advanced Computation
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801

February 1973

This work was supported in part by the Advanced Research Projects Agency of the Department of Defense and was monitored by the U.S. Army Research Office-Durham under Contract No. DAHC04-72-C-0001.

TABLE OF CONTENTS

	Page
PREFACE	1
ERROR ANALYSIS	2
FUNCTION EVALUATION AND COMPLEXITY THEORY	3
APPROXIMATION	6
LINEAR ALGEBRA	9
NONLINEAR EQUATIONS	17
QUADRATURE	18
ORDINARY DIFFERENTIAL EQUATIONS	19
PARTIAL DIFFERENTIAL EQUATIONS	21
INTEGRAL EQUATIONS	23
INTERVAL AND SIGNIFICANT DIGIT ARITHMETIC	23
OPTIMIZATION	24
GRAPH ALGORITHMS	27
KEY TO REPORT SOURCES	30

PREFACE

In our efforts to keep abreast of developments in computational mathematics, we have found it useful to make this listing of abstracts of the many departmental reports which we have accumulated during the past year.

The abstracts are largely those provided with the documents; where no abstract was given we used selected sentences from the introduction. Occasionally we condensed an abstract which seemed too long for our purposes. Sources of the reports are identified by letter codes prefacing the report number. A key to these codes is given at the end of the listing.

We wish to thank Nancy Freece not only for her excellent and rapid job of typing but also for doing a considerable amount of the work of hunting through the documents to assemble the information listed.

ERROR ANALYSIS

Algorithms to Reveal Properties of Floating-Point Arithmetic
by Michael A. Malcolm

Two algorithms are presented in the form of FORTRAN subroutines. Each subroutine computes the radix and number of digits of the floating-point numbers and whether rounding or chopping is done by the machine on which it is run. The methods are shown to work on any "reasonable" floating-point computer.

STAN-CS-71-211 March 1971 8 Pages Program Incl.

On the Precision Attainable with Various Floating-Point Number Systems
by R. P. Brent

For scientific computations on a digital computer the set of real numbers is usually approximated by a finite set F of "floating-point numbers". We compare the numerical accuracy possible with different choices of F having approximately the same range and requiring the same wordlength. In particular, we compare different choices of base (or radix) with the usual floating-point systems. The emphasis is on the choice of F, not on the details of the number representation or the arithmetic, but both rounded and truncated arithmetic are considered. Theoretical results are given, and some simulations of typical floating-point computations (forming sums, solving systems of linear equations, finding eigenvalues) are described. If the leading fraction bit of a normalized base-2 number is not stored explicitly (saving a bit), and the criterion is to minimize the mean square roundoff error, then base 2 is best. If unnormalized numbers are allowed, so the first bit must be stored explicitly, then base 4 (or sometimes base 8) is the best of the usual systems.

YORK-RC 3751 February 1972 28 Pages

Performance Statistics for the FORTRAN IV (H) and PL/1 (Version 5) Libraries in IBM OS/360 Release 18
by Kenneth E. Hillstrom

The computational subroutine libraries associated with the FORTRAN IV (H) and PL/I (F) (Version 5) compilers of OS Release 18 have been tested. The testing techniques are described and accuracy and timing statistics are presented; proper operation under error conditions was verified.

ANL-7666 August 1970 70 Pages Program Incl.

FUNCTION EVALUATION AND COMPLEXITY THEORY

On The Number of Multiplications for the Evaluation of a Polynomial and Some of its Derivatives
by Mary Shaw and J. F. Traub

Some recent work in computational complexity has dealt with the number of arithmetic operations needed to evaluate a polynomial or a polynomial and its first derivative. Here we consider the evaluation of a polynomial and its first m derivatives and, in particular, the calculation of all the derivatives.

CMU - No # August 1972 18 Pages

Computational Complexity of Iterative Processes
by J. F. Traub

The theory of optimal algorithmic processes is part of computational complexity. This paper deals with analytic computational complexity. The relation between the goodness of an iteration algorithm and its new function evaluation and memory requirements are analyzed. A new conjecture is stated.

CMU-CS-71-105 October 1971 25 Pages

On the Additions Necessary to Compute Certain Functions
by David G. Kirkpatrick

We describe low level or functional complexity, fitting it into the general framework of computational complexity and presenting a number of results in arithmetic complexity. We put forward a new notion of independence, called rational independence, which applies to the terms of a computed expression. We are able to show that this notion serves as a measure for the number of additions needed to compute a function.

TOR-Tech. Rpt. 39 February 1972 75 Pages [M.S. Thesis]

Some Results in the Study of Algorithms
by J. Ian Munro

Functions such as polynomials, matrix products, and digraph transitive closures are in practice computed very often. In recent years, considerable attention has been turned to efficient computation of such functions, with the hope of minimizing the time needed to compute them on a general purpose digital computer as we now know it.

In this thesis several new algorithms are given for the evaluation of such functions, and in some cases these are shown to be optimal under a reasonable model of computation.

It is shown that if a polynomial, or set of polynomials, is to be evaluated at a large number of points, and these points are all given at the same time, then the number of arithmetic operations needed per "polynomial-point" is less than linear in the degree of the polynomial.

An algorithm requiring work of the order of max (n log n,e) is given for the determination of the strongly connected components of an n-node, e-edge digraph.

Attention is turned to problems of computing arithmetic functions under a model of computation in which a large degree of parallelism is permitted. New optimal and almost optimal algorithms are presented for several such computations.

TOR-Tech. Rpt. 32 October 1971 102 Pages Thesis

Application of Continued Fractions for Fast Evaluation of Certain Functions on a Digital Computer
by Amnon Bracha

The purpose of this paper is to develop a method for evaluation of certain elementary functions on a digital computer by the use of continued fractions. The time required for this evaluation is drastically reduced by using "short" operations like shift and add, instead of multiplications. Functional consistency is the most important factor that allows the expansion of a function into a continued fraction. Several cases are discussed and in particular the solution of the quadratic equation is discussed in more detail to demonstrate the convergence of the method.

UIUC-R-72-510 March 1972 29 Pages

Bounds on Polynomial Evaluation Algorithms
by Larry Joseph Stockmeyer

The purpose of this work is to investigate the number of arithmetic operations required by algorithms which evaluate polynomials. Previous results show that a polynomial of degree n requires at least n/2 multiplication/divisions and at least n addition/subtractions for its evaluation if the coefficients of the polynomial are suitably independent irrational numbers. However, the coefficients of any polynomial that would be evaluated in practice are represented only to a finite accuracy and are therefore rational numbers. The above results

are extended to show that the same lower bounds hold for almost all rational polynomials if the polynomial is being evaluated efficiently. Another lower bound result is given that shows that almost all rational polynomials of degree n require at least $\sqrt{n}$ multiplication/divisions for their evaluation by any algorithm, efficient or not.

Several algorithms are presented which can in theory evaluate any rational polynomial using $O(\sqrt{n})$ multiplications and many additions. While of no practical use for rational polynomials in general, these algorithms do turn out to give methods for evaluating a polynomial at a matrix argument which are more efficient than previous methods.

MAC TR-98 April 1972 54 Pages

APPROXIMATION

Variational Study of Nonlinear Spline Curves
by E. H. Lee and G. E. Forsythe

This is an exposition of the variational and differential properties of nonlinear spline curves, based on the Euler-Bernoulli theory for the bending of thin beams or elastica. For both open and closed splines through prescribed nodal points in the euclidean plane, various types of nodal constraints are considered, and the corresponding algebraic and differential equations relating curvature, angle, arc length, and tangential force are derived in a simple manner. The results for closed splines are apparently new, and they cannot be derived by the consideration of a constrained conservative system. There is a survey of the scanty recent literature.

STAN-CS-229-71 August 1971 26 Pages

An Algorithm for Fitting Related Sets of Straight-Line Data
by Geneva G. Belford

Many physical experiments give rise to sets of curves related by the requirement that, although certain of the curve parameters may vary from curve to curve, others should be the same for all of the curves. To get the "best" values of the common parameters, one would like to fit all of the curves simultaneously by the appropriate theoretical expressions. This paper deals with this problem, presenting an algorithm for its solution, in the case that the curves are straight lines with common slope and "best" fit is defined in the uniform (or minimax) sense.

CAC No. 54 December 1972 12 Pages

An Implementation of the Remes Algorithm for Minimax Approximation
by Roland Olofsson

The ALGOL procedure for the Remes algorithm given by G. H. Golub and L. B. Smith (Comm ACM 14 (1971), 737-746 Alg 414) has been implemented and tested on a CD 3200 computer. Some examples of ill conditioned generalized polynomials have been used to test the algorithm. A comparison is also made to another program based on the Polya algorithm.

UMINF-20.72 April 1972 14 Pages

On the Existence and Characterization of Minimal Projections
by P. D. Morris and E. W. Cheney

A projection from a normed linear space X onto a subspace Y is a bounded linear operation P: $X \to Y$ having the property that $Py = y$ for all $y \in Y$. Projections play an important role in numerical analysis and approximation theory. The use of projections is based upon the acceptance of Px as an approximation of x in the subspace Y. The quality of approximations produced by P depends upon $\|P\|$ and $\|I-P\|$. The present work is devoted mainly to the characterization problem of minimal projections.

CNA-37 January 1972 32 Pages

Stability Properties of Trigonometric Interpolating Operators
by P. D. Morris and E. W. Cheney

We consider the familiar process of trigonometric interpolation with 2n+1 equally-spaced nodes. This process is interpreted here as a linear projection operator P acting on the space C of all real, continuous, 2π-periodic functions. The range of this projection P is the subspace Π of all trigonometric polynomials of order n. Many other useful projections exist which map C onto Π, and attention here is focused on their extremal properties.

CNA-51 August 1972 20 Pages

Extremal Properties of Approximation Operators
by K. H. Price and E. W. Cheney

In 1910, de La Vallee Poussin published some researches on the following approximation problem: given the continuous function $x = x(t)$ defined on the interval $[-1,1]$, and given n+1 points $t_0,\ldots,t_n$ in $[-1,1]$, find the polynomial p of degree < n which minimizes $\max_{0\leq i\leq n} |x(t_i) - p(t_i)|$.

The polynomial p depends linearly on x. In this respect, the process resembles Lagrange interpolation; both processes define linear projection operators.

In this paper we study operators of this type, which depend upon n+1 quanta of information about the function x. We examine the family of all such operators, and seek out those that are optimal in some sense.

CNA-54 August 1972 22 Pages

A Generalization of the Construction Method of Laurent and Carasso for Interpolating L-Splines
by M. J. Munteanu

The purpose of this study is to generalize the method given by P. T. Laurent and C. Carasso for the construction of ordinary polynomial spline interpolation functions.

We explain in Section 2 in detail the extension of the method to the case of polynomial spline functions interpolating in the sense of Hermite. In Section 3 we treat briefly the generalization to interpolating L-splines. Finally, in Section 4, we give a numerical example to illustrate certain aspects of the method.

CNA-11 February 1971 17 Pages

Computation of Smoothing and Interpolating Natural Splines via Local Bases
by Tom Lyche and Larry L. Schumaker

The purpose of this paper is twofold. First we show how smoothing splines can be represented in terms of local support basis splines, and how the corresponding coefficients can be computed by solving a linear system whose matrix is banded. This leads to an efficient and accurate algorithm for computing smoothing splines. Secondly, we extend some ideas of de Boor to obtain recursion relations for the basis functions which allows their computation as positive combinations of positive quantities.

CNA-17 April 1971 21 Pages

On a Method of Carasso and Laurent for Constructing Interpolating Splines
by M. J. Munteanu and L. L. Schumaker

Carasso and Laurent studied a method for computing natural polynomial splines interpolating simple data. We discuss several similar methods which can be applied to numerical construction of more general interpolating splines, including Lg-splines interpolating Extended-Hermite-Birkhoff data.

CNA-40 April 1972 20 Pages

Direct and Inverse Theorems for Multidimensional Spline Approximation
by M. J. Munteanu and L. L. Schumaker

A general approximation theoretic result of Butzer and Scherer is used to obtain direct and inverse theorems as well as saturation theorems for multidimensional spline approximations of functions in certain Sobolev spaces.

CNA-43 April 1972 16 Pages

LINEAR ALGEBRA

On the Matrix Polynomial, Lambda-Matrix and Block Eigenvalue Problems
by J. E. Dennis, Jr., J. F. Traub, and R. P. Weber

A matrix S is a solvent of the matrix polynomial $M(X) \equiv X^m + A_1X^{m-1} + \ldots + A_m$, if $M(S) = \underline{\underline{0}}$, where A_i, X and S are square matrices. We present some new mathematical results for matrix polynomials, as well as a globally convergent algorithm for calculating such solvents.

In the theoretical part of this paper, existence theorems for solvents, a generalized division, interpolation, a block Vandermonde, and a generalized Lagrangian basis are studied. Algorithms are presented which generalize Traub's scalar polynomial methods, Bernoulli's method, and eigenvector powering.

The related lambda-matrix problem, that of finding a scalar λ such that

$$I\lambda^m + A_1\lambda^{m-1} + \ldots + A_m$$

is singular, is examined along with the matrix polynomial problem.

The matrix polynomial problem can be cast into a block eigenvalue formulation as follows. Given a matrix A of order mn, find a matrix S of order n, such that AV = VX, where V is a matrix of full rank. Some of the implications of this new block eigenvalue formulation are considered.

CMU-CS-71-110 December 1972 117 Pages Programs Incl.

The QL-Algorithm for computing the eigenvalues of a skewsymmetric matrix
by Axel Ruhe

Two ALGOL procedures are presented which compute the eigenvalues and, if desired, the eigenvectors of a real skewsymmetric matrix. The first reduces the matrix into tridiagonal form by the method of Householder, and the second uses the QL-algorithm to compute the eigenvalues of the tridiagonal matrix. The procedures take advantage of the skewsymmetry, and work considerably faster than a code applicable to general matrices.

UMINF-16.72 January 1972 17 Pages Programs Incl.

The QR algorithm to find the eigensystem of a skewsymmetric matrix
by Bo Kågstrom

The QR algorithm is specialized in order to deal effectively with a real skewsymmetric matrix. The algorithm starts with Householder

tridiagonalization, and uses a double shift QR method to get the purely imaginary eigenvalues, entirely using real arithmetic. Different strategies are compared, on a couple of numerical examples.

UMINF-14.71 December 1971 14 Pages

Multiple step gradient iterative methods for computing eigenvalues of large symmetric matrices
by Britt Marie Alsén

Solving the eigenproblem for the matrix A, can be thought of as extremizing the Rayleigh quotient.

$$R(x) = \frac{x^H A x}{x^H x} .$$

Application of an optimization algorithm to R(x) finds one of the end eigenvalues. In the present contribution algorithms are tested, which extremize R(x) simultaneously over m orthogonal vectors, thus finding m eigenvalues. A version of the conjugate gradient algorithm and steepest descent are tried on a couple of numerical examples, mainly such emanating from difference approximations.

UMINF-15.71 December 1971 19 Pages

Simultaneous inverse iteration for computing orthogonal eigenvectors
by Anders Isacsson

Inverse iteration is an accurate and fast method to find eigenvectors of a symmetric matrix, where the eigenvalues are computed by bisection or the QR-algorithm. If the matrix has a cluster of eigenvalues, the vectors computed are however not orthogonal. In the present contribution an algorithm is developed which iterates simultaneously on a set of vectors, and thereafter orthogonalizes them. This algorithm produces orthogonal eigenvectors to a real symmetric matrix having an arbitrary distribution of eigenvalues.

UMINF-19.72 March 1972 20 Pages

Quasi-Newton algorithms applied to eigenvalue problems
by Johnny Widén

In the present contribution the eigenproblem (linear or nonlinear), (1) $T(\lambda)x = 0$, is solved by seeking local minima of the functional, (2) $\|T(\lambda)x\|_2$ by means of quasi-Newton algorithms.

The experimental evidence suggests that this is not a feasible approach to the problem, since even in well conditioned cases, the minimization of (2) is difficult numerically, because several values of λ and vectors x give the function (2) values of equally small magnitudes.

UMINF-23.72 April 1972 25 Pages

Simultaneous iteration for finding eigenvalues of a symmetric matrix
by Lennart Lindström

The algorithm of simultaneous iteration as described by Bauer and Rutishauser is tested numerically on some test matrices.

UMINF-13.71 November 1971 10 Pages

MIDAS - Solution of Linear Algebraic Equations
by P. A. Businger

The information contained in this abstract should be sufficient to use the MIDAS package to solve general nonsingular systems of linear algebraic equations, invert matrices, and compute determinants. ("MIDAS" stands for Matrix, Inverse, Determinant, and Algebraic System). The computational method, nonstandard calling sequences, and an evaluation of the program are presented in the memorandum. Error bounds on the solution or inverse are available as an option.

BTL No # January 1970 41 Pages Programs Incl.

The Rotation of Eigenvectors by a Perturbation - III
by Chandler Davis and W. M. Kahan

When a hermitian linear operator is slightly perturbed, by how much can its invariant subspaces change? Given some approximations to a cluster of neighbouring eigenvalues and to the corresponding eigenvectors of a real symmetric matrix, and given an estimate for the gap that separates the cluster from all other eigenvalues, how much can the subspace spanned by the eigenvectors differ from the subspace spanned by our approximations? These questions are closely related; both are investigated here. The difference between the two subspaces is characterized in terms of certain angles through which one subspace must be rotated in order most directly to reach the other.

TOR Tech. Rpt. 6 November 1968 103 Pages

Inclusion Regions for Partitioned Matrices
by Douglas Dale Olesky

A region in the complex plane containing all of the eigenvalues of a complex matrix A is said to be an inclusion region. The object of this thesis is to define algorithms for the computation of inclusion regions by partitioning the matrix A.

TOR Tech. Rpt. 31 September 1971 Thesis 117 Pages

Square Roots of an Orthogonal Matrix
by Erold W. Hinds

This thesis is concerned with the problem of determining numerically the positive stable orthogonal square root Q of a given real rotation A, that is, a rotation Q such that $Q^2 = A$ and $Q+Q'$ is positive semi-definite. Detailed accounts of two particular algorithms which may be used to construct such half-rotations are presented herein.

TOR Tech. Rpt. 37 December 1971 Thesis 167 Pages Programs Incl.

On a Characterization of the Best ℓ_2 Scaling of a Matrix
by G. H. Golub and J. M. Varah

This paper is concerned with best two-sided scaling of a general square matrix, and in particular with a certain characterization of that best scaling: namely that the first and last singular vectors (on left and right) of the scaled matrix have components of equal modulus. Necessity, sufficiency, and its relation with other characterizations are discussed. Then the problem of best scaling for rectangular matrices is introduced and a conjecture made regarding a possible best scaling. The conjecture is verified for some special cases.

UBC Tech. Rpt. 72-08 12 Pages

On the Solution of Block Tridiagonal Systems Arising from Certain Finite-Difference Equations
by J. M. Varah

UBC Tech. Rpt. 72-02 March 1972 17 Pages

On the Numerical Solution of Ill-Conditioned Linear Systems with Applications to Ill-Posed Problems
by J. M. Varah

UBC Tech. Rpt. 71-02 October 1971 21 Pages

Parallel Computation of Eigenvalues of Real Matrices
by David J. Kuck and Ahmed Sameh

This paper describes the implementation of three standard matrix eigenvalue computation methods on an array machine with high efficiency. A brief description of the ILLIAC IV computer is provided as background material. Three major sections follow--the first two describe Jacobi and Householder algorithms for real symmetric matrices, and the third describes the QR algorithm for real nonsymmetric matrices. Each of these sections is divided into four parts. The theoretical background of the method is presented first. An ILLIAC IV implementation of the algorithm is presented and timing estimates are included.

Next, the efficiency (ratio of number of computations actually performed to number of computations possible in a given time) of the computation on an array machine is derived for primary memory constrained matrices. Finally, eigenvalue computations for matrices too large to be contained in primary memory are discussed in terms of a head-per-track secondary storage device.

It is shown that for Jacobi and Householder algorithms, parallel computation efficiencies of 90% are possible, while those of the QR algorithm are rather low.

CAC No. 9 November 1971 39 Pages

Calculating the Eigenvectors of Diagonally Dominant Matrices
by M. M. Blevins and G. W. Stewart

This paper proposes an algorithm for calculating the eigenvectors of a diagonally dominant matrix all of whose elements are known to high relative accuracy. Eigenvectors corresponding to pathologically close eigenvalues are treated by computing the invariant subspace that they span. If the off-diagonal elements of the matrix are sufficiently small, the method is superior to standard techniques, and indeed it may produce a complete set of eigenvectors with an amount of work proportional to the square of the order of the matrix. An analysis of the effects of perturbations in the matrix on the eigenvectors is given.

CNA-47 June 1972 25 Pages

Error and Perturbation Bounds for Subspaces Associated with Certain Eigenvalue Problems
by G. W. Stewart

This paper describes a technique for obtaining error bounds for certain characteristic subspaces associated with the algebraic eigenvalue problem, the generalized eigenvalue problem, and the singular value decomposition. The method also gives perturbation bounds for isolated eigenvalues and useful information about clusters of eigenvalues. The bounds are obtained from an iterative process for generating the subspaces in question, and one or more steps of the iteration can be used to construct perturbation estimates whose error can be bounded.

CNA-46 June 1972 34 Pages

A Note on Non-Hermitian Perturbations of Hermitian Matrices
by G. W. Stewart

Let A and E be Hermitian matrices. It is well known that the eigenvalues α_i of A and β_i of A + E can be put into a one-one correspondence in such a way that $|\alpha_i - \beta_i| \leq \|E\|_2$ and $\sum(\alpha_i - \beta_i)^2 \leq \|E\|_F^2$.

In this note similar results are established for a class of non-Hermitian perturbations.

CNA-41 April 1972 4 Pages

Conjugate Direction Algorithms and Matrix Decompositions
by G. W. Stewart

The notion of a set of directions conjugate with respect to a matrix A is generalized and applied to the solution of the linear system $Ax = b$. Variants of an algorithm for producing conjugate directions are shown to yield several standard matrix decompositions, such as the LU and QR factorizations of A, as well as a number of previously studied iterations for solving linear systems. The application of these results to systems of nonlinear equations and to least squares problems is discussed.

CNA-49 July 1972 16 Pages

Linear Stationary Second-Degree Methods for the Solution of Large Linear Systems
by David M. Young and David R. Kincaid

In this paper we consider a class of linear stationary second-degree methods for solving the linear system $Au = b$. The second-degree methods which we shall consider are related to a "basic" linear stationary method of first degree of the form $u^{(n+1)} = Gu^{(n)} + k$ where G is a matrix such that I-G is nonsingular and $k = (I-G)A^{-1}b$. These second-degree methods are of the form

$$u^{(1)} = p_1 Gu^{(0)} + q_1 u^{(0)} + w^{(1)}$$

$$u^{(n+1)} = pGu^{(n+} + qu^{(n)} + ru^{(n-1)} + w, \quad n = 1, 2, \ldots., .$$

CNA-52 August 1972 29 Pages

Error Bounds for Approximate Subspaces of Closed Linear Operators in Hilbert Space
by G. W. Stewart

It is well known to specialists in matrix computations that the eigenvectors of a matrix corresponding to a set of poorly separated eigenvalues are quite sensitive to perturbations in the elements of the matrix. It is also known that for Hermitian matrices the invariant subspace corresponding to a cluster of eigenvalues is insensitive to such perturbations. It is the object of this paper to extend such results to nonnormal matrices and, more generally, to closed operators in Hilbert space.

CNA-4 October 1970 25 Pages

On the Numerical Properties of an Iteration for Computing the Moore-Penrose Generalized Inverse
by G. W. Stewart

In this paper some of the numerical problems associated with computing the generalized inverse of a matrix are discussed and illustrated by a detailed analysis of an iteration of Ben-Israel and Cohen.

CNA-12 March 1971 27 Pages

On the Solution of Large Systems of Linear Algebraic Equations with Sparse, Positive Definite Matrices
by David M. Young

The object of this paper is to discuss the current status of iterative methods for solving large systems of linear algebraic equations. Primary emphasis is on those systems involving sparse matrices where iterative methods appear more attractive than direct methods.

CNA-55 August 1972 45 Pages

The Solution of Large Systems of Linear Algebraic Equations by Iterative Methods
by David M. Young

In this paper we provide a summary of some of the available iterative methods for solving large systems of linear algebraic equations. We are primarily concerned with the case where the matrix of the system is "sparse", i.e., has only a few non-zero elements in each row. Systems of this kind typically arise in the solution by finite difference methods of elliptic partial differential equations:

CNA-19 May 1971 23 Pages

An Algorithm for the Generalized Matrix Eigenvalue Problem $Ax = \lambda Bx$
by C. B. Moler (Stanford University) and G. W. Stewart

A new method, called the QZ algorithm, is presented for the solution of the matrix eigenvalue problem $Ax = \lambda Bx$ with general square matrices A and B. Particular attention is paid to the degeneracies which result when B is singular. No inversions of B or its submatrices are used. The algorithm is a generalization of the QR algorithm, and reduces to it when $B = I$. A Fortran program and some illustrative examples are included.

CNA-32 October 1971 50 Pages
and STAN-CS-242-71 Programs Incl.

A Generalization of the LR Algorithm to Solve $Ax = \lambda Bx$
by Linda Kaufman

In this paper, we will present and analyze an algorithm for finding $\underset{\sim}{x}$ and λ such that $A\underset{\sim}{x} = \lambda B\underset{\sim}{x}$ where A and B are n x n matrices. The algorithm does not require matrix inversion, and may be used when either or both matrices are singular. Our method is a generalization of Rutishauser's LR method for the standard eigenvalue problem $A\underset{\sim}{x} = \lambda\underset{\sim}{x}$ and closely resembles the QZ algorithm given by Moler and Stewart for the generalized problem given above. Unlike the QZ algorithm, which uses orthogonal transformations, our method, the LZ algorithm, uses elementary transformations. When either A or B is complex, our method should be more efficient.

STAN-CS-72-276 April 1972 72 Pages Program Incl.

Richardson's Non-Stationary Matrix Iterative Procedure
by R. S. Anderssen and G. H. Golub

Because of its simplicity, Richardson's non-stationary iterative scheme is a potentially powerful method for the solution of (linear) operator equations. However, its general application has more or less been blocked by (a) the problem of constructing polynomials, which deviate least from zero on the spectrum of the given operator, and which are required for the determination of the iteration parameters of the non-stationary method, and (b) the instability of this scheme with respect to rounding error effects.

STAN-CS-72-304 August 1972 73 Pages

Bidiagonalization of Matrices and Solution of Linear Equations
by C. C. Paige

An algorithm given by Golub and Kahan for reducing a general matrix to bidiagonal form is shown to be very important for large sparse matrices. The singular values of the matrix are those of the bidiagonal form, and these can be easily computed. The bidiagonalization algorithm is shown to be the basis of important methods for solving the linear least squares problem for large sparse matrices. Eigenvalues of certain 2-cyclic matrices can also be efficiently computed using this bidiagonalization.

STAN-CS-72-295 June 1972 28 Pages

An Efficient Parallel Algorithm for the Solution of a Tridiagonal Linear System of Equations
by Harold S. Stone

Tridiagonal linear systems of equations can be solved on conventional serial machine in a time proportional to N, where N is the number of equations. The conventional algorithms do not lend them-

selves directly to parallel computation on computers of the ILLIAC IV class, in the sense that they appear to be inherently serial. An efficient parallel algorithm is presented in which computation time grows as $\log_2 N$. The algorithm is based on recursive doubling solutions of linear recurrence relations, and can be used to solve recurrence relations of all orders.

STAN-CS-71-251 December 1971 21 Pages

NONLINEAR EQUATIONS

Error Bounds, Based Upon Gerschgorin's Theorems for the Zeros of a Polynomial
by Brian Thomas Smith

Suppose L distinct points $z_1, z_2, \ldots z_L$ are given in the complex plane and associated with each point z_k is a positive integer M_k, the 'multiplicity' of z_k; These points z_k, with their multiplicities M_k, are supposed to approximate the zeros of a monic polynomial P(z). The object of this thesis is to measure how well a set of such points approximates the zeros of P(z); that is, we want to determine how close each point z_k is to its subset of M_k zeros.

TOR Tech. Rpt. 9 July 1969 137 Pages [Ph.D. Thesis]

An Algorithm for the Solution of a Quadratic Equation using Continued Fractions
by Kishor Shridharbhai Trivedi

This is an effort to investigate representations of numbers other than positional notation for computer arithmetic. Using continued fraction representation of numbers, an algorithm to solve a limited class of quadratics has been developed. This algorithm is suitable for hardware implementation and is reasonably efficient. Feasibility of constructing an arithmetic unit with continued fraction representation depends on discovery of many more such useful algorithms which can share the same hardware.

UIUC-R-72-525 June 1972 62 Pages Program [M.S. Thesis]

A Method for Solving Polynomial Equations by Continued Fractions
by Amnon Bracha

A method for the approximation of all the real roots of an n-order polynomial equation is developed. It is assumed that intervals containing the solutions are known. Bilinear transformations are used to approximate the solution. Convergence is achieved.

UIUC-R-72-521 July 1972 20 Pages

Kublanovskayas algorithm for solving the nonlinear eigenvalue problem
by Per Lindström

In the present contribution an iterative algorithm proposed by B. N. Kublanovskaya (Dokl Akad Nauk 188, 1004-1005 (1969) compare also SIAM Journal of Num. Analysis 7, 532-537 (1970)) for the solution of the nonlinear eigenvalue problem $T(\lambda)x = 0$ is tested numerically. $T(\lambda)$ is an $n \times n$ real or complex matrix that depends analytically on the eigenvalue parameter λ. It is our aim to study different possible algorithms for this problem in order to assess their practical value, and this is a part of this investigation.

UMINF-5.71 August 1971 14 Pages

Algorithms for the nonlinear eigenvalue problem
by Axel Ruhe

The following nonlinear eigenvalue problem is studied: Let $T(\lambda)$ be an $n \times n$ matrix, whose elements are analytical functions of the complex number λ. Seek λ and vectors x and y, such that $T(\lambda)x = 0$ and $y^H T(\lambda) = 0$. Several algorithms for the numerical solution of this problem are studied. These algorithms are extensions of algorithms for the linear eigenvalue problem such as inverse iteration and the QR algorithm, and algorithms that reduce the nonlinear problem into a sequence of linear problems. Numerical tests, performed in order to compare the different algorithms, are reported, and a few numerical examples illustrating their behaviour are given. An ALGOL program for one of the algorithms is given as an appendix.

UMINF-7.71 August 1971 30 Pages Program

QUADRATURE

Multi-Dimensional Quadrature Formulae
by P. Keast

Two classes of formulae, for numerical quadrature in several variables, are considered. Comparisons are made on the basis of accuracy and efficiency of the various formulae used, and on the basis of ease of obtaining similar formulae in high dimension. A new technique for obtaining a particular type of quadrature rule is developed. Several numerical examples are given.

TOR Tech. Rpt. 40 February 1972 78 Pages Program

ORDINARY DIFFERENTIAL EQUATIONS

Studies in the Numerical Solution of Stiff Ordinary Differential Equations
by Wayne Enright

A class of second derivative multi-step formulas is developed. The stability of these formulas is investigated and they are shown to be suitable for stiff equations. An implementation of a variable-order, variable-step method based on this class of formulas is described and some numerical results are presented.

The reliability of methods suitable for stiff equations is also investigated. The condition of α-acceptability is introduced and it is shown that if a method satisfies this condition, then for any member of a particular class of stiff systems, the numerical solution can be guaranteed to lie on the exact solution of a slightly perturbed system.

Methods based on the second derivative formulas mentioned above are shown to be α-acceptable. Numerical comparisons of methods in terms of both reliability and efficiency are considered and the background is established for a comprehensive comparison.

TOR Tech. Rpt. 46 October 1972 82 Pages Program [Ph.D. Thesis]

On the Identification of Multi-Output Linear Time-Invariant and Periodic Dynamic Systems
by Ahmed H. Sameh and Walter L. Heimerdinger

In this paper we describe an efficient computational algorithm for estimating the coefficients of the characteristic polynomial of a linear time-invariant multi-output dynamic system, using only output observations, for qualitative analysis of the transition matrix or for evaluating its eigenvalues. We also give some computational results of the identification of those systems using the Ho-Kalman approach. Furthermore, an identification scheme for high-frequency periodic systems of unknown periods is described in detail.

CAC No. 53 November 1972 29 Pages

Tchebycheffian Multistep Methods for Ordinary Differential Equations
by Tom Lyche

The purpose of this paper is to extend some of the theory for linear multistep methods to include stepsize-dependent coefficients. In particular we treat the case where we demand exact integration of a given set of linearly independent functions.

CNA-16 April 1971 20 Pages

Numerical Methods for the Identification of Differential Equations
by Raymond Jonathan Lermit

This dissertation considers computational methods designed to aid in mathematical model building. Specifically, it discusses methods of determining ordinary differential equations given their solution in the form of observed data.

Since the problem cannot be solved in this generality, it is necessary to supply equations containing arbitrary functions. The problem is then to find these functions given the solution of the equations. In order to be amenable to computer solution, a discretization of the functions as a linear sum of a given orthonormal set is necessary. The problem thus reduces to one of finding a finite number of parameters.

The solution technique is to find that function which produces a solution most closely approximating the observed data. It is thus a problem in the minimization of nonlinear functionals and may be solved by iterative methods. Modifications required to ensure that the functional is convex, thus guaranteeing a global minimum solution, are discussed. Different algorithms considered for carrying out the minimization include the generalization of Newton's method and variants of it which are more economical in computer time, especially the Conjugate Gradient method. All of these methods require derivatives which are derived automatically from the original equation using formal algebraic manipulation. The different methods are compared for rates of convergence and amount of calculation required at each iteration.

Two examples are included. The effect of introducing random errors into the data to simulate observational errors, and how this may alter the convergence rates, is also discussed.

CAC No. 49 June 1972 84 Pages Program [Ph.D. Thesis]

PARTIAL DIFFERENTIAL EQUATIONS

Finite-Element Galerkin Method for Mixed Initial-Boundary Value Problems in Elasticity Theory
by Hiroshi Fujii

The Finite-Element Galerkin method (FEG method) has been recognized as a very powerful tool for numerical solution of boundary value problems of partial differential equations. In this paper, we treat the application of the FEG method to vibration problems (i.e., mixed initial-boundary value problems) in linear elasticity theory. We introduce some special versions of FEG schemes for approximating the solution of our problem, including schemes of explicit type as well as an implicit scheme and a continuous-time scheme.

CNA-34 October 1971 70 Pages

Automatic Solutions of Partial Differential Equations
by Leonard Andrew Larsen

The problem of the present paper is to look for an automatic method that can be applied to some subset of partial differential equations. The primary value of a program involving an automatic method would be to provide the person who needs only a few runs with a particular type of equation the opportunity to find a solution, within acceptable tolerances, without having to write and debug a specialized program.

UIUC-R-72-546 October 1972 128 Pages Program [Ph.D. Thesis]

An Application of Semi-Iterative and Second-Degree Symmetric Successive Overrelaxation Iterative Methods
by Tran Phien

The goal of this paper is to study the effectiveness of semi-iterative methods and second-degree methods based on the symmetric successive overrelaxation method for solving a certain class of linear systems. The linear systems correspond to the solution by a finite difference method of a boundary value problem associated with a particular self-adjoint elliptic partial differential equation. We shall use a semi-iterative method based on the symmetric successive overrelaxation (SSOR) method. The resulting method is called the SSOR-SI method. We shall also use the SSOR-SD method, a stationary second-degree method based on the SSOR method.

CNA-42 May 1972 48 Pages [M.S. Thesis]

Numerical Implementation of the Schwarz Alternating Procedure for Elliptic Partial Differential Equations
by David Ross Stoutemyer

This thesis describes numerical implementation of the Schwarz and Neumann alternating procedures for the solution of the Laplace-Dirichlet problem on the union of two disks, the intersection of two disks, an arbitrary quadrilateral, and the union of two spheres.

All of the above examples lead to a pair of coupled Fredholm integral equations of the second kind with singular kernels and singular low-order derivatives of the solution at corners of the region. These singularities are overcome by a change of variable together with special spacing of the abscissas and extrapolation to the limit. These methods are suitable for more general boundary conditions, more general partial differential equations, and more general geometrical configurations.

STAN-CS-72-283 May 1972 131 Pages Program [Thesis]

Use of Fast Direct Methods for the Efficient Numerical Solution of Nonseparable Elliptic Equations
by Paul Concus and Gene H. Golub

We study an iterative technique for the numerical solution of strongly elliptic equations of divergence form in two dimensions with Dirichlet boundary conditions on a rectangle. The technique is based on the repeated solution by a fast direct method of a discrete Helmholtz equation on a uniform rectangular mesh. The problem is suitably scaled before iteration, and Chebyshev acceleration is applied to improve convergence. We show that convergence can be exceedingly rapid and independent of mesh size for smooth coefficients. Extensions to other boundary conditions, other equations, and irregular mesh spacings are discussed, and the performance of the technique is illustrated with numerical examples.

STAN-CS-72-278 April 1972 39 Pages

Numerical Solution of First-Order Hyperbolic Systems of Partial Differential Equations
by Sunil K. Pal

This work develops two new finite-difference schemes - an explicit scheme and an implicit scheme - for numerical solution of first-order hyperbolic systems of partial differential equations in any number of space variables.

TOR Tech. Rpt. 13 1969 94 Pages [Ph.D. Thesis]

INTEGRAL EQUATIONS

Methods for the Numerical Solution of Integral Equations of the Second Kind
by David Blair Coldrick

A detailed analysis of the quadrature method is given from several points of view, viz. those of Kantorovich, Buckner, and the Mysovskih-Brakhage-Anselone development. It is shown that the technique of deferred approach to the limit is valid under fairly general circumstances. An error analysis of projection (and "shifted" projection)methods is given. A class of degenerate kernel methods is proposed, and is compared to closely related projection methods in terms of error bounds and ease of implementation. Finally, a discussion of the problem of (weak) singularities is presented. The application of projection methods to such equations is mentioned. The technique of product integration is applied to this problem, and a fairly general convergence statement is established. For singularities the kernel k(s, t) along the line s = t, a generalization of Kantorovich's method with a higher order of convergence is proposed, and a theorem to this effect is proved.

TOR Tech. Rpt. 45 October 1972 152 Pages [Ph.D. Thesis]

INTERVAL AND SIGNIFICANT DIGIT ARITHMETIC

Implementation of Basic Software for Significant Digit Arithmetic
by Steven See Sun Lai

This report is concerned with the implementation of significant digit arithmetic using the unnormalized arithmetic of Metropolis and Ashenhurst. One of the primary objectives was to design the basic software modules for inclusion into the OL/2 array language; however, these modules are written in assembly language and therefore are adaptable to other software systems for the IBM 360/370 machines. A discussion of significance arithmetic, including the nontrivial problem of input/output, is presented. Examples are provided in Appendix A.

UIUC-R-72-530 June 1972 76 Pages [Thesis] Program Incl.

A Univac 1108 Program for Obtaining Rigorous Error Estimates for Approximate Solutions of Systems of Equations
by Dennis Kuba and L. B. Rall

A UNIVAC 1108 computer program which obtains rigorous interval error bounds for approximate solutions of finite systems of nonlinear equations is described in this report. Since the coefficients of the original system may take on interval values, the error bounds obtained include the contributions of truncation error for Newton's method, round-off error, and possible errors in the coefficients of the given system of equations.

MRC Tech. Rpt. 1168 January 1972 155 Pages Program Incl.

OPTIMIZATION

Large-Scale Linear Programming Using the Cholesky Factorization
by M. A. Saunders

A variation of the revised simplex method is proposed for solving a standard linear programming problem. The method is derived from an algorithm recently proposed by Gill and Murray, and is based upon the orthogonal factorization $B = LQ$ or, equivalently, upon the Cholesky factorization $BB^T = LL^T$ where B is the usual square basis, L is the lower triangular and Q is orthogonal.

We wish to retain the favorable numerical properties of the orthogonal factorization, while extending the work of Gill and Murray to the case of linear programs which are both large and sparse. The principal property exploited is that the Cholesky factor L depends only on <u>which</u> variables are in the basis, and not upon the <u>order</u> in which they happen to enter. A preliminary ordering of the rows of the full data matrix therefore promises to ensure that L will remain sparse throughout the iterations of the simplex method.

An initial (in-core) version of the algorithm has been implemented in Algol W on the IBM 360/91 and tested on several medium-scale problems from industry (up to 930 constraints). While performance has not been especially good on problems of high density, the method does appear to be efficient on problems which are very sparse, and on structured problems which have either generalized upper bounding, block-angular, or staircase form.

STAN-CS-72-252 January 1972 60 Pages

Product Form of the Cholesky Factorization for Large-Scale Linear Programming
by Michael A. Saunders

A variation of Gill and Murray's version of the revised simplex algorithm is proposed, using the Cholesky factorization $BB^T = LDL^T$ where B is the usual basis, D is diagonal and L is unit lower triangular. It is shown that during change of basis L may be updated in product form. As with standard methods using the product form of inverse, this allows use of sequential storage devices for accumulating updates to L. In addition the favorable numerical properties of Gill and Murray's algorithm are retained.

Close attention is given to efficient out-of-core implementation. In the case of large-scale block-angular problems, the updates to L will remain very sparse for all iterations.

STAN-CS-72-301 August 1972 38 Pages

The Differentiation of Pseudoinverses and Nonlinear Least Squares Problems Whose Variables Separate
by G. H. Golub and V. Pereyra

For given data (t_i, y_i), $i = 1, \ldots, m$, we consider the least squares fit of nonlinear models of the form

$$F(\underset{\sim}{a}, \underset{\sim}{\alpha}; t) = \sum_{j=1}^{n} g_j(\underset{\sim}{a}) \phi_j(\underset{\sim}{\alpha}; t), \underset{\sim}{a} \varepsilon \mathbb{R}^s, \underset{\sim}{\alpha} \varepsilon \mathbb{R}^k.$$

For this purpose we study the minimization of the nonlinear functional

$$r(\underset{\sim}{a}, \underset{\sim}{\alpha}) = \sum_{i=1}^{m} (y_i - F(\underset{\sim}{a}, \underset{\sim}{\alpha}, t_i))^2.$$

It is shown that by defining the matrix$\{\Phi(\underset{\sim}{\alpha})\}_{i,j} = \phi_j(\underset{\sim}{\alpha}; t_i)$ and the modified functional $r_2(\underset{\sim}{\alpha}) = \|\underset{\sim}{y} - \Phi(\underset{\sim}{\alpha})\Phi^+(\underset{\sim}{\alpha}) \underset{\sim}{y}\|_2^2$, it is possible to optimize first with respect to the parameters $\underset{\sim}{\alpha}$, and then to obtain, a posteriori, the optimal parameters $\hat{\underset{\sim}{a}}$. The matrix $\Phi^+(\underset{\sim}{\alpha})$ is the Moore-Penrose generalized inverse of $\Phi(\underset{\sim}{\alpha})$, and we develop formulas for its Fréchet derivative under the hypothesis that $\Phi(\underset{\sim}{\alpha})$ is of constant (though not necessarily full) rank. From these formulas we readily obtain the derivatives of the orthogonal projectors associated with $\Phi(\underset{\sim}{\alpha})$, and also that of the functional $r_2(\underset{\sim}{\alpha})$. Detailed algorithms are presented which make extensive use of well-known reliable linear least squares techniques, and numerical results and comparisons are given.

STAN-CS-72-261 February 1972 49 Pages Program Incl.

The method of conjugate gradients used in iverse iteration
by Axel Ruhe and Torbjörn Wiberg

An algorithm is devised that improves an eigenvector approximation corresponding to the largest (or smallest) eigenvalue of a large and sparse symmetric matrix. It solves the linear systems that arise in inverse iteration by means of the c-g algorithm. Stopping criteria are developed which insure an accurate result, and in many cases give convergence after a small number of c-g steps.

UMINF-22.72 April 1972 21 Pages

Partial Analysis of an Algorithm for Minimizing Functions Based on a Homogenous Model
by James W. Daniel

Jacobson and Oksman recently presented a method which they claimed minimized a homogeneous function of arbitrary degree in finitely many steps and which was globally convergent for a large class of general functions. We point out here errors in the proofs of these results, suggest some modifications in the method so that the global convergence result is valid, and we give some examples to show the computational effects of these changes.

CNA-15 April 1971 14 Pages

Convergent Step Sizes for Curvilinear-Path Methods of Minimization
by James W. Daniel

When one seeks a point minimizing a function f over a set C--which may be the whole space--one often moves from one approximate solution x_n to another $x_{n+1} = x_n + t_n p_n$ by searching along a ray $x_n + tp_n$, where the step-size t_n must be chosen. More generally one can consider moving along a curvilinear path $x_n + p_n(x_n,t)$. The present paper shows how the usual step-size algorithms can be used with this general approach.

CNA-29 July 1971 24 Pages

Nonlinear Least Squares Without Derivatives: An Application of the QR Matrix Decomposition
by Richard H. Bartels

A new algorithm is proposed for minimizing sums of squares of nonlinear functions of several variables without the use of derivatives. The algorithm is constructed using a multivariate secant technique (Broyden rank-one method) to approximate the Jacobian of the summands, and this Jacobian is used to drive the Levengerg-Marquardt iteration. The Jacobian is kept and updated in QR decomposition, and the linear least squares subproglems which arise at each iteration cycle are solved by Golub's method. A FORTRAN program and test results are offered. It is shown that the algorithm accomodates linear equality constraints with ease. Remarks are made concerning the treatment of rank deficiencies in Golub's method with respect to algorithms of the type presented.

CNA-44 April 1972 47 Pages Program Incl.

Global Convergence for Newton Methods in Mathematical Programming
by James W. Daniel

In constrained optimization problems in mathematical programming one wants to minimize a functional f(x) over a given set C. If, at an approximate solution x_n, one replaces f(x) by its Taylor series expansion through quadratic terms at x_n and denotes by x_{n+1} the minimizing point for this over C, one has a direct analogue of Newton's method. The local convergence of this has been previously analyzed; here we give global convergence results for this and the similar algorithm in which the constraint set C is also linearized at each step.

CNA-48 June 1972 10 Pages

Newton's Method for Nonlinear Inequalities
by James W. Daniel

A Newton-type algorithm has been presented elsewhere for solving non-linear inequalities of the form $f(x) \leq 0$, and quadratic convergence has been proved under very strong hypotheses. In this paper we show that the same results hold under a considerable weakening of the hypotheses.

CNA-53 August 1972 10 Pages

GRAPH ALGORITHMS

Graph Isomorphism
by Derek Gordon Corneil

A procedure for determining whether two graphs are isomorphic is described. During the procedure, from any given graph two graphs, the representative graph and the reordered graph, are derived.

The time required to determine both derived graphs depends on a power of n, the order of the given graph. This power is a function of an adjacency property known as the strong regularity of the given graph. For graphs that do not contain a strongly regular transitive subgraph, the power is, at worst, five.

All given timing estimates for graphs that do not contain a strongly regular transitive subgraph are confirmed. The algorithm has been programmed and in the implemented version one of the following four messages will come out: (1) The graphs are isomorphic. (2) The graphs are not isomorphic. (3) The representative graphs are identical; the reordered graphs are not identical; hence, these graphs form a counter-example to the conjecture. (4) The graphs contain a 3-strongly regular subgraph.

TOR Tech. Rpt. 18 April 1970 [Ph.D. Thesis] Program

An Algorithm to Determine the Chromatic Number of a Graph
by Barry Graham

A heuristic algorithm for the determination of the chromatic number of a finite graph is presented. This algorithm is based on Zykov's theorem for chromatic polynomials and extensive empirical tests show that it is the best algorithm available. Christofides' algorithm for the determination of chromatic number is described and is used in the comparison tests. An Algol-W coding of both algorithms is included in the appendix.

TOR Tech. Rpt. 47 November 1972 82 Pages [M.S. Thesis] Program

---

Algorithms for Finding Cliques of a Graph
by Gordon D. Mulligan

Various methods for determining cliques in undirected graphs are presented and analyzed. Testing schemes to compare the methods on graphs with a maximum number of cliques and on graphs that attempt to represent some applications are described.

A theorem states that the maximum number of cliques in a graph increases exponentially as the number of vertices increases. For this reason heuristics are employed in clique finding algorithms to decrease the amount of search and an attempt is made to measure the algorithms as a function of the number of cliques.

Three algorithms are formally defined and tested: the Bierstone algorithm, the Bron-Kerbosch algorithm, and the Corneil algorithm. The Bron-Kerbosch algorithm is judged to be the best.

TOR Tech. Rpt. 41 May 1972 89 Pages [M.S. Thesis] Program

Spectra of Finite Graphs
by L. Collatz, Hamburg and U. Sinogowitz, Darmstadt

Various problems encountered in practical applications, such as approximations of characteristic frequencies of a membrane with fixed perimeter and given area, or calculations of air vibrations in a bounded volume using difference methods, gave rise to the consideration of graphs in general.

In this paper finite (in a subsequent paper certain types of infinite) graphs are examined and in particular some conclusions drawn with regard to their relationship to the theory of indecomposable, non-negative matrices.

UAE No. 10 February 1968 28 Pages

Graphs, Groups and Matrices
by Abbe Mowshowitz

Our object here is to exploit the connection between the adjacency matrix of a graph and its automorphism group in order to determine the latter.

In what follows, we will take advantage of the fact that the adjacency matrix of a graph is a (0, 1)-matrix and thus can be regarded as a matrix over GF(2).

UBC Tech. Rpt. 71-04 October 1971 14 Pages

Combinatorial Solutions to Partitioning Problems
by J. A. Lukes

In this dissertation we describe algorithms that use graph properties and dynamic programming techniques to generate the optimal partition of an arbitrary graph. In particular, let G be a graph with weighted nodes and weighted edges. We consider algorithms that solve the problem of partitioning G into clusters of nodes such that the sum of the node weights in any cluster does not exceed a given maximum W and the weights of the intercluster edges are minimized. An interesting application of such an algorithm is the assignment of a program's subroutines and data to pages in a paged memory system so as to minimize paging faults.

A very efficient variation of the general algorithm results if the graph to be partitioned is a tree. We show that trees can be partitioned in a time proportional to the number of nodes in the graph.

STAN-CS-72-293 May 1972 120 Pages [Ph.D. Thesis]

Chromatic Automorphisms of Graphs
by V. Chvatal and J. Sichler

The coloring group and the full automorphism group of an n-chromatic graph are independent if and only if n is an integer ≥ 3.

STAN-CS-72-273 March 1972 11 Pages

KEY TO REPORT SOURCES

ANL	-	Argonne National Laboratory Argonne, Illinois 60439
BTL	-	Bell Telephone Laboratories, Inc. Murray Hill, New Jersey 07971
CAC	-	Center for Advanced Computation University of Illinois, Urbana, Illinois 61801
CMU	-	Carnegie-Mellon University Computer Science Department Pittsburgh, Pennsylvania 15213
CNA	-	Center for Numerical Analysis University of Texas, Austin, Texas 78712
MAC	-	Massachusetts Institute of Technology Project MAC Cambridge, Massachusetts 02139
MRC	-	Mathematics Research Center University of Wisconsin, Madison, Wisconsin 53706
STAN	-	Computer Science Department Stanford University, Stanford, California 94305
TOR	-	Department of Computer Science University of Toronto, Ontario, Canada
UAE	-	Department of Computing Science University of Alberta, Edmonton, Alberta, Canada
UBC	-	Department of Computer Science University of British Columbia, Vancouver, B. C., Canada
UMINF	-	Department of Information Processing University of Umea, Umea, Sweden
UIUC	-	Department of Computer Science University of Illinois, Urbana, Illinois 61801
YORK	-	T. J. Watson Research Center Yorktown Heights, New York 10598

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Center for Advanced Computation University of Illinois at Urbana-Champaign Urbana, Illinois	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

Computational Mathematics Abstracts

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Bibliographical Abstracts

5. AUTHOR(S) *(First name, middle initial, last name)*

Edited by: Geneva Belford, Jonathan Lermit, George Purdy, and Ahmed Sameh

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
February 1973	35	

8a. CONTRACT OR GRANT NO. DAHC04-72-C-0001 b. PROJECT NO. ARPA Order No. 1899 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) CAC Document No. 64 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT

Copies may be requested from the address given in (1) above.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	U. S. Army Research Office-Durham Duke Station, Durham, North Carolina

13. ABSTRACT

NONE

DD FORM 1 NOV 65 1473

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Numerical Analysis Graph Theory Mathematical Programming						